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Suction/injection effects on thermophoresis particle deposition in a non-Darcy porous medium under the influence of Soret, Dufour effects

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ABSTRACT

The effect of suction/injection on thermophoretic particle deposition in free convection on a vertical plate embedded in a fluid saturated non-Darcy porous medium is studied using similarity solution technique. The effect of Soret and Dufour parameters on convective transport, wall thermophoretic deposition velocity, heat transfer and mass transfer is discussed in detail for different values of dispersion parameters, (Ra_γ, Ra_ξ) inertial parameter F and Lewis number Le. The result indicates that in both suction, injection the Soret effect is more influential in increasing the concentration distribution in both aiding as well as opposing buoyancies. Also, it is worth mentioning here that the combined effect of opposing buoyancy and injection will have a more significant effect on the boundary layer thickness. In both the cases, suction as well as injection, magnitude of heat transfer is observed to be more when the second order effects are considered than when they are not. But, mass transfer and the wall thermophoretic deposition velocity V_{tw} becomes less when all effects are considered than when they are not.

1. Introduction

Convective flow through porous media is an area of research undergoing rapid growth in the fluid mechanics and heat transfer field due to its broad range of scientific and engineering applications. It is associated with petroleum and geothermal processes, fiber and granular insulation materials, high performance insulation buildings, transpiration cooling, packed bed chemical reactors, control of pollutant spread in ground water. A nice review about heat transfer in geothermal system has been presented in Cheng [1].

Thermophoresis is a phenomenon by which submicron-sized particles suspended in a non-isothermal gas acquires a mean speed relative to the gas in the direction of decreasing temperature. It is a mechanism for the capture of particles on cold surfaces. In many industries the composition of the processing gas may contain any of an unlimited range of particle, or gaseous contaminants and may be influenced by uncontrolled factors of temperature and humidity. When such an impure gas is *injected* on a solid surface, a boundary layer will develop, and energy and momentum transfer give rise to temperature and velocity gradients. Mass transfer caused by gravitation, molecular diffusion, eddy diffusion and inertial impact results in deposition of the suspended components onto the surface. This phenomenon has got outstanding importance in vast number of applications. A few of them are: in determining exhaust gas particle trajectories from combustion devices, in studying the particulate material deposition on turbine blades. Also this has been sited as a cause for the deposition of particulate matter on heat exchange surfaces with the attendant reduction of the heat transfer coefficient. It is an established fact that thermophoresis is the dominant mass transfer mechanism in the modified chemical vapor deposition (MCVD) process used in the fabrication of optical fiber performs. Thermophoresis is also significant in view of its relevance to postulated accidents by radioactive particle deposition in nuclear reactors. In variety of applications, there is a need to predict the transport rates of aerosol particles. This phenomenon is used in air cleaning and aerosol sampling devices. The detailed study regarding practical applications of thermophoretic phenomenon can be found in [2,3].

A paper by Epstein et al. [4] deals with the thermophoretic deposition in natural convection flow from a vertical plate. But the analysis was considered for the cold surface. Garg and Jayaraj [5] analyzed numerically the thermophoretic deposition of small particles due to impingement of a laminar slot jet on an inclined plate using an implicit finite difference scheme. The analysis was done for the cold, hot and adiabatic plate conditions. The study of thermophoresis particle deposition on a vertical plate was extended to porous medium by Chamkha and Pop [6]. Thermophoresis particle deposition under different situations can be seen in [7,8,30–33].

The effect of blowing and suction along a vertical flat plate on free convection in air or water has been the subject of numerous investigators. The stability of the boundary layer is considerably

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Nomenclature

C	dimensional concentration
c	inertial coefficient
C_s	concentration susceptibility
C_p	specific heat at constant pressure
d	pore diameter
D	constant molecular diffusivity
D_e	effective solutal diffusivity
f_w	positive-suction
f_w	negative-injection
k	thermophoretic constant
k_T	thermal diffusion ratio
k_g	thermal conductivity of the gas
k_p	thermal conductivity of the particle
K	permeability of the porous medium
R_p	particle radius
T	dimensional temperature
U	dimensional velocity component along X direction
V	dimensional velocity component along Y direction

Greek symbols

α	constant thermal diffusivity
α_e	effective thermal diffusivity
β_T	coefficient of thermal expansion
β_C	coefficient of solutal expansion
γ	coefficient of thermal dispersion
ν	fluid kinematic viscosity
λ	mean free path length
ρ	fluid density
μ	viscosity of the fluid
η	similarity variable
ψ	dimensional stream function
θ	non-dimensional temperature
ϕ	non-dimensional concentration
ξ	coefficient of solutal dispersion
θ_w	$=T_w - T_\infty$
ϕ_w	$=C_w - C_\infty$

Parameters

$\Pr = \frac{\nu}{\alpha}$	Prandtl number
$Le = \frac{\alpha}{D}$	diffusivity ratio
$N = \frac{\beta_C \phi_w}{\beta_T \theta_w}$	buoyancy ratio
$F = \frac{c\sqrt{K} K g \beta_T \theta_w}{\nu^2}$	inertial Parameter
$Ra_d = \frac{K g \beta_T \theta_w d}{\alpha \nu}$	pore-dependent Rayleigh number
$Ra_\xi = \xi Ra_d$	solutal dispersion parameter
$Ra_\gamma = \gamma Ra_d$	thermal dispersion parameter
$V_t = \left(\frac{-k\Pr}{N_t+\theta}\right)\frac{\partial \theta}{\partial y}$	non-dimensional thermophoretic velocity
$D_f = \frac{D k_T \phi_w}{C_s C_p \alpha \theta_w}$	parameter representing Dufour effect
$S_r = \frac{D k_T \theta_w}{C_s C_p \alpha \phi_w}$	parameter representing Soret effect
$V_{tw} = \frac{-k\Pr}{1+N_t}\theta'(0)$	non-dimensional wall thermophoretic deposition velocity

Subscripts

w	evaluated on the wall
∞	evaluated at the outer edge of the boundary layer

influenced by continuous suction and blowing. Suction or blowing causes a double effect with respect to heat transfer. On the one hand temperature profile is influenced by the changed velocity field in the boundary layer leading to a change in the heat conduction at the wall. On the other hand convective heat transfer occurs at the wall along with heat conduction for various values of suction/injection parameter. Using the similarity solution procedure for different wall temperature variations and for different injection/suction velocity distributions, Cheng [9] studied the natural convection heat transfer from a vertical wall in a Darcian fluid saturated porous medium. Agarwal and Roy [10] showed that the velocity of the liquid increases with increase in the suction velocity and decreases with increase in the injection velocity. The analysis about the influence of suction/injection on heat and mass transfer is discussed in [11–15].

The most important application of blowing (Schlichting and Gersten [16]) is in so-called transpiration cooling. If the blown fluid is different from the outer flow a binary boundary layer arises. As well as heat and momentum exchanges there is also exchange of mass through diffusion. Since blowing out a light gas (Gross et al. [17]) drastically reduces the heat transfer this measure is used in practice in heat protection. Very light gases – helium and hydrogen (Baron and Scott [18]) have a particularly good cooling effect. This cooling process may lead to clogging. In some fluid flow systems (Cebeci and Bradshaw [19]) warm air is blown along the surface to raise the local air temperature and to evaporate water droplets and films. In the gas turbine, in response to the constant demand for the increased cycle temperature that result in increased thermal efficiency a stream of relatively cold gas is injected along the inner surface of the combustors and along the external surface of guide vanes and blades to create a layer that insulates the wall from hot gases. Verms [20] has studied the deposition rates in cooled and uncooled turbines cascades. It was found that temperature difference between the wall and the gas could cause a 15-fold increase in deposition rate as compared with the case of adiabatic cascade. Gokoglu and Rosner [21] studied the effect of particulate thermophoresis in reducing the fouling rate advantage of diffusion cooling in gas turbines. Correlation has been developed to predict thermophoretically enhanced diffusion deposition rates including the effect of transpiration cooling.

Electronic components (Cengel [22]) placed in an enclosure are cooled by natural convection by providing sufficient number of vents to enable the cooled air to enter and the heated air to leave the case. When natural convection cooling is not adequate, air is blown with out taking into account the exact need. In this process all kinds of contaminants that are present in the air, such as lint, dust moisture and even oil are deposited on the surface. These contaminants can pile up on the components and plug up narrow passage ways, causing over heating. It should be remembered that the dust that settles on the electronic components acts as an insulation layer that makes it very difficult for the heat generated in the components to escape. In order to minimize the deposition of contaminants on the surface, the volume flow rate of the air into the surface is to be controlled. This can be achieved by calculating accurately the heat and mass transfer at the surface taking into account the effect of thermophoresis.

Based on the concept of high-mass transfer with a blowing parameter a simple approach for evaluating the effect of wall suction and thermophoresis on aerosol particle deposition from a laminar flow over a flat plate has been discussed by Tsai [23]. The situation of primary interest in this problem is a cooled wall immersed in a hot fluid stream which can be considered to be a model problem for deposition of aerosol particles from a condensing superheated vapor. Such a wall will capture some of small particles by the mechanisms of Brownian diffusion, convection with suction or blowing, and thermophoresis. The technological problems include particle deposition onto a surface from a condensing vapor–gas mixture, a semi-conductor wafer in the electronic industry. Alam et al. [24] have discussed the effect of variable suction and thermophoresis on steady MHD combined free-forced convective heat and mass transfer flow over a semi-infinite permeable inclined plate in the presence of thermal radiation.

Mixing and recirculation of local fluid streams occur as the fluid moves through tortuous paths in packed beds. This hydrodynamic mixing of fluid at pore level causes thermal and solutal dispersion in porous medium. This becomes more considerable for moderate and fast flows. The book by Nield and Bejan [25] provides more discussions and applications of convective transport in porous media. Detailed discussion and literature survey is available in Murthy [26].

In industrial and chemical engineering processes which involve multi-component fluid, concentrations vary from point to point resulting in mass transfer. The Soret and Dufour effects are very significant when the temperature and the concentration gradients are high. Heated jets or diffusion flames created by blowing combustible gas from a vertical pipe are controlled by forced convection in the initial region and by buoyancy forces far from the jet or pipe exist. Industrial smokestacks usually have a significant momentum flux to assist the initial rise of the contaminant plume. The simplest physical model of such a flow is two-dimensional laminar flow along a vertical flat plate. Recent applications of this model can be found in the area of reactor safety, combustion flames and solar collectors as well as building energy conservation. Bourich and Hasnaoui [27] studied analytically and numerically the Soret effect on the onset of convection in a vertical porous layer subjected to uniform heat flux. The influence of Soret and Dufour effects on flow field in free convection boundary layer from a vertical surface embedded in a Darcian porous medium has been studied by Postelinicu [28] and Anghel et al. [29].

The Soret and Dufour effects and thermophoresis particle deposition becomes more significant when the concentration gradients, temperature gradients are high. Also, the inertial effect, dispersion effects, suction/injection effect has a significant contribution to convective transport in porous medium. Certainly, the combined effect of these parameters will have large impact on heat and mass transfer rates. The applications such as transpiration cooling, chemical engineering processes which involve multi-component fluid, are not discussed taking into consideration, suction/injection effects on thermophoresis. Therefore accurate prediction of the non-dimensional heat and mass transfer coefficients and concentration at the wall is not at all possible by neglecting Soret, Dufour, Suction/Injection, thermophoresis effects. Partha [31] has analyzed the thermophoresis particle deposition in a non-Darcy porous medium under the influence of Soret and Dufour effects. Hence in this article, we analyze Suction/Injection effects on thermophoresis particle deposition in a non-Darcy porous medium under the influence of Soret and Dufour effects.

2. Governing equations

Consider the problem of non-Darcy natural convection heat and mass transfer from a semi-infinite vertical surface embedded in a fluid saturated non-Darcy porous medium (Fig. 1). The wall is maintained at constant temperature and concentration T_w and C_w, respectively. The ambient medium temperature and concentration are T_∞ and C_∞, respectively. The X-axis is taken along the plate and the Y-axis is normal to it. We assume that the fluid and the porous medium have constant physical properties. The isothermal hot wall is assumed to be permeable with a lateral mass flux in the form $\upsilon_w = Ax^l$ where $x = 0$ represents the leading edge of the hot wall. Here, $\upsilon_w = 0$ corresponds to the impermeable wall case.

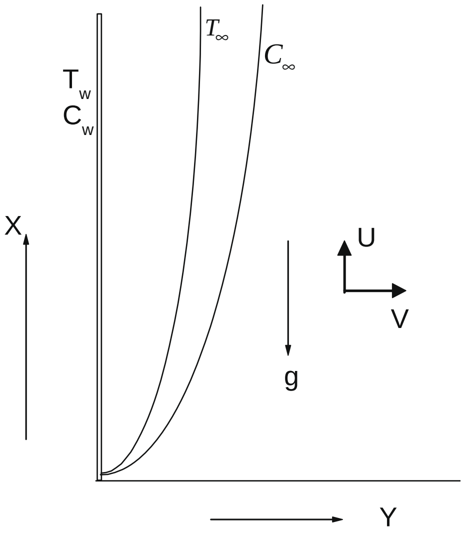

Fig. 1. Schematic drawing of the problem.

The fluid flow is moderate and the permeability of the medium is low so that the Forchheimer flow model is applicable. With the Boussinesq approximation, the governing equations for the boundary layer flow from the wall to the fluid saturated porous medium can be written as

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{1}$$

$$U + \frac{c\sqrt{K}}{\nu}U^2 + \frac{Kg}{\nu}[\beta_T(T - T_\infty) + \beta_C(C - C_\infty)] \tag{2}$$

$$U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y} = \frac{\partial}{\partial Y}\left(\alpha_e\frac{\partial T}{\partial Y}\right) + \frac{Dk_T}{C_sC_P}\frac{\partial^2 C}{\partial Y^2} \tag{3}$$

$$U\frac{\partial C}{\partial X} + V\frac{\partial C}{\partial Y} + \frac{\partial}{\partial Y}(C\upsilon_t) = \frac{\partial}{\partial Y}\left(D_e\frac{\partial C}{\partial Y}\right) + \frac{Dk_T}{C_sC_P}\frac{\partial^2 T}{\partial Y^2} \tag{4}$$

along with the boundary conditions

$$Y = 0, \quad V = V_w(x) = Ax^L, \quad T = T_w, \quad C = C_w \tag{5}$$

$$Y \to \infty, \quad U \to 0, \quad T \to T_\infty, \quad C \to C_\infty \tag{6}$$

Here U and V are the velocity components along X and Y directions, T is the temperature, C is concentration, c is the inertial coefficient, K is the permeability constant, β_T and β_C are the coefficients of thermal and solutal expansions, ν is the kinematic viscosity, ρ is the density, g is the acceleration due to gravity, α_e is the effective thermal diffusivity and D_e is the solutal diffusivity. The thermal and solutal dispersion diffusivities can be written as $\alpha_e = \alpha + \gamma dU$, $D_e = D + \xi dU$, where γ and ξ are coefficients of thermal and solutal dispersions, respectively, α and D are constant thermal and molecular diffusivities, respectively. k_T is the thermal diffusion ratio, C_s is concentration susceptibility and C_P is the specific heat at constant pressure.

The temperature gradient established in the thermal boundary layer drives the particle either towards the plate where they get deposited or away from the plate thereby forming a critical layer adjacent to the plate. The velocity acquired by the small particles relative to the gas velocity is related to the temperature gradient in the flow field is given by

$$\upsilon_t = -k(\nu/T)\nabla T \tag{7}$$

Here ν is the kinematic viscosity of the gas and k is the thermophoretic coefficient. Here we assume that the thermophoretic constant k is independent of temperature. Due to the boundary layer behavior the temperature gradient $\frac{\partial T}{\partial Y}$ is much larger than $\frac{\partial T}{\partial X}$. From Eq. (7) it follows that the component of v_t normal to the plate is given by $v_t = -k(v/T)\frac{\partial T}{\partial Y}$. It is assumed that in the absence of thermophoresis the particles move with the fluid at the local fluid velocity.

Eqs. (1)–(4) are transformed into ordinary differential Eqs. (8)–(10) with the dimensionless numbers and similarity parameters defined in the paper by Chamkha and Pop [6].

$$f'' + 2Ff'f'' = (\theta' + N\phi') \tag{8}$$

$$\theta'' + \frac{1}{2}f\theta' + Ra_\gamma(f'\theta'' + f''\theta') + D_f\phi'' = 0 \tag{9}$$

$$\phi'' + \frac{Le}{2}f\phi' + LeRa_\xi(f'\phi'' + f''\phi') + \frac{Lek\mathrm{Pr}}{N_t + \theta}\left[\theta'\phi' + \phi\theta'' - \frac{\phi}{N_t + \theta}\theta'^2\right] + S_r Le\theta'' = 0 \tag{10}$$

The boundary conditions are

$$\eta = 0: \quad f = f_w, \quad \theta = 1, \quad \phi = 1 \tag{11}$$

$$\eta \to \infty: \quad \theta \to 0, \quad \phi \to 0, \quad f' \to 0 \tag{12}$$

On the wall ($\eta = 0$) the velocity component v becomes $v_w = \frac{-\alpha}{2x}Ra_x^{1/2}f_w$ and the particular value of L for which v_w will be free from x is $L = -1/2$. Hence, with this value of L the boundary conditions also become free from x. Thus the resulting ordinary differential equations with the boundary conditions can be solved using the general techniques.

In the above $F = \left(\frac{c\sqrt{K}Kg\beta_T\theta_w}{v^2}\right)$ is the inertial Parameter, $Ra_\gamma = \gamma Ra_d$, $Ra_\xi = \xi Ra_d$ represents thermal and solutal dispersion respectively, $Ra_d = \frac{Kg\beta_T\theta_w d}{\alpha v}$ is the pore diameter-dependent Rayleigh number, $Le = \frac{\alpha}{D}$ is the diffusivity ratio (Lewis number), $N = \frac{\beta_c\phi_w}{\beta_T\theta_w}$ is the buoyancy ratio, $V_t = \left(\frac{-k\mathrm{Pr}}{N_t+\theta}\right)\frac{\partial\theta}{\partial y}$ is the thermophoretic velocity, $N_t = \frac{T_w - T_\infty}{T_\infty}$. Here D_f and S_r are the non-dimensional parameters representing Dufour and Soret effect given by $D_f = \frac{Dk_T\phi_w}{C_sC_p\alpha\theta_w}$, $S_r = \frac{Dk_T\theta_w}{C_sC_p\alpha\phi_w}$. The parameter $N > 0$ represents the aiding buoyancy and $N < 0$ represents the opposing buoyancy. The heat transfer and mass transfer coefficient in their non-dimensional form are written as

$$Nu_x/Ra_x^{1/2} = -[1 + Ra_\gamma f'(0)]\theta'(0) \tag{13}$$

$$Sh_x/Ra_x^{1/2} = -[1 + Ra_\xi f'(0)]\phi'(0) \tag{14}$$

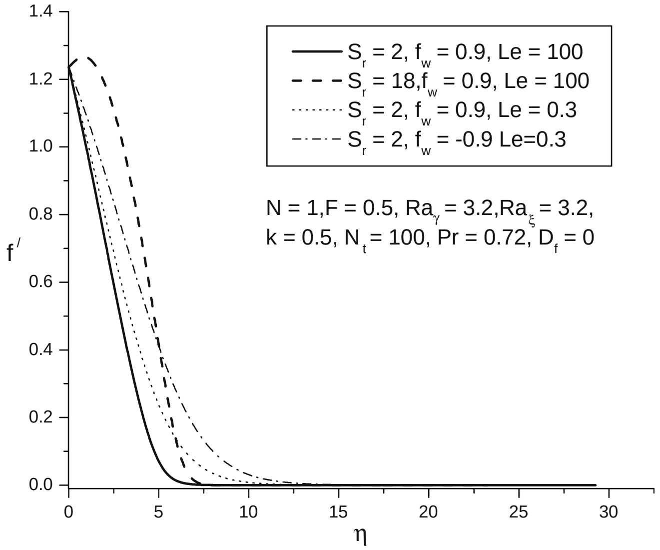

Fig. 2. Effect of S_r and Le on tangential velocity f' in suction and injection.

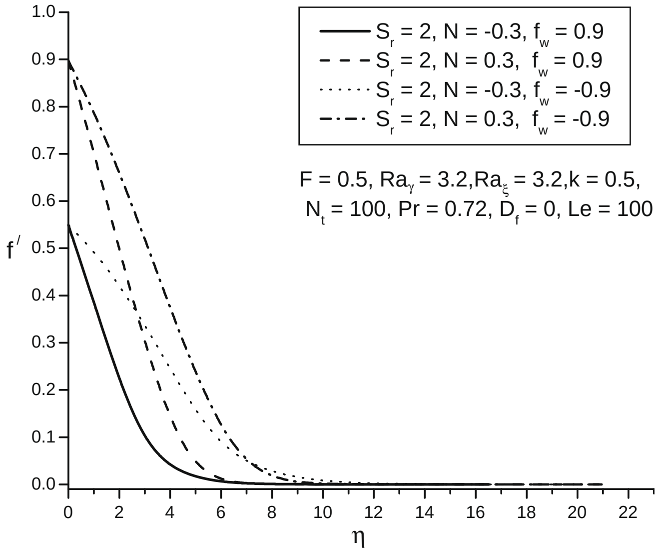

Fig. 3. Effect of aiding and opposing buoyancies on tangential velocity f' in suction and injection for fixed value of S_r.

3. Results and discussion

The ordinary differential Eqs. (8)–(10) along with the boundary conditions (11 and 12) are integrated by giving appropriate initial guess values for $f'(0), \theta'(0)$ and $\phi'(0)$ to match the values with the corresponding boundary conditions $f'(\infty), \theta'(\infty)$ and $\phi'(\infty)$, respectively. Nag software (DO2HAEF routine) is used for integrating the corresponding first-order system of equations and for shooting and matching the initial boundary conditions. The integration length η_∞ varies with parameter values and it has been chosen suitably every time such that the boundary conditions at the outer edge of the boundary layer are satisfied. The results obtained here are accurate up to fourth decimal place. Extensive calculations have been performed with different values of parameters to obtain the flow, temperature, concentration fields inside the boundary layer.

Of interest in this problem are the non-dimensional concentration profiles $\phi(\eta)$ and wall thermophoretic deposition velocity V_{tw} given by $V_{tw} = \frac{-k\mathrm{Pr}}{1+N_t}\theta'(0)$. Hence the effect of the parameters D_f and S_r on the concentration distribution, wall thermophoretic deposition velocity V_{tw}, non-dimensional heat transfer coefficient and non-dimensional mass transfer coefficient is studied in detail for different values of the parameters in aiding as well as opposing buoyancies for both suction and injection. In order to understand the significance of second order effects the parameters are given non-zero values and zero values in both the cases of suction and injection.

Figs. 2–4 represent the flow field for various values of Le, D_f and S_r. From Fig. 2 we observe that the velocity boundary layer thickness increases as the value of S_r increases. Also, it clearly indicates that as we proceed from the suction zone to injection zone (f_w = 0.9 to −0.9) the velocity boundary layer thickness increases, thus indicating that injection is more effective on the flow field. Also, we notice that for the smaller values of Le, the boundary layer thickness is larger. In both suction and injection, where S_r = 2, (Fig. 3) the velocity boundary layer thickness is seen to be more in the case of aiding buoyancy than in the case of opposing buoyancy. The conclusion drawn from Fig. 2 that the thickness of the boundary layer is broadened due to injection is spotted again in Fig. 3. The same is highlighted in Fig. 4 where the parameter D_f is given the value 2. Thus it is comprehensible that injection is significant irrespective of the parameter values.

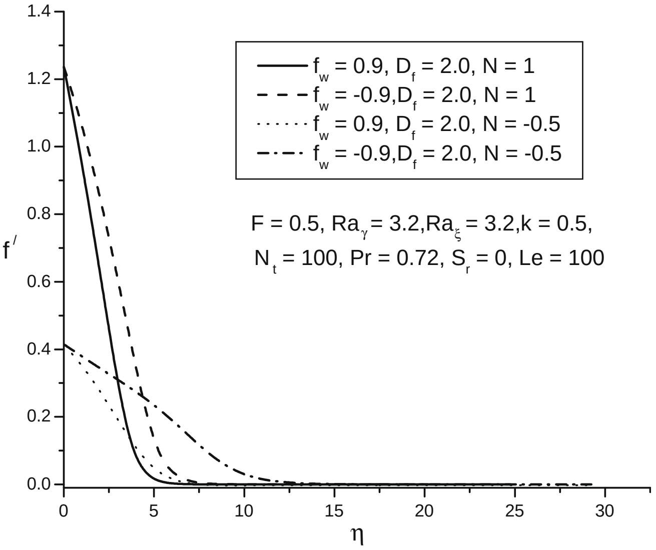

Fig. 4. Effect of aiding and opposing buoyancies on tangential velocity f' in suction and injection for fixed value of D_f.

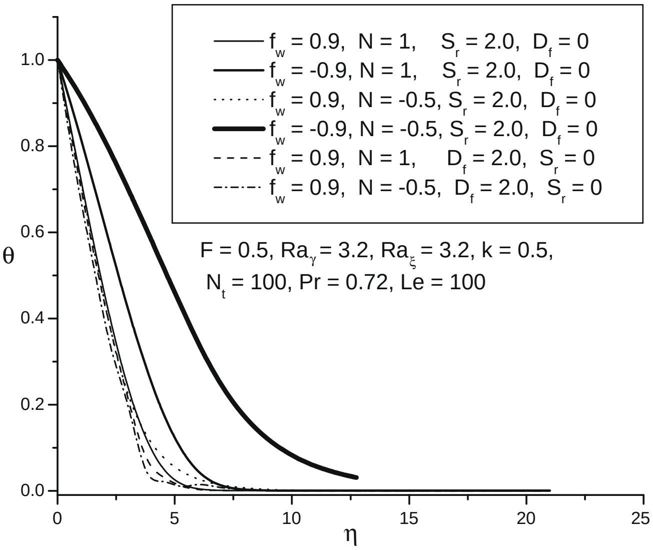

Fig. 5. Effect of Suction, Injection on temperature distribution for fixed values of D_f and S_r in aiding and opposing buoyancies.

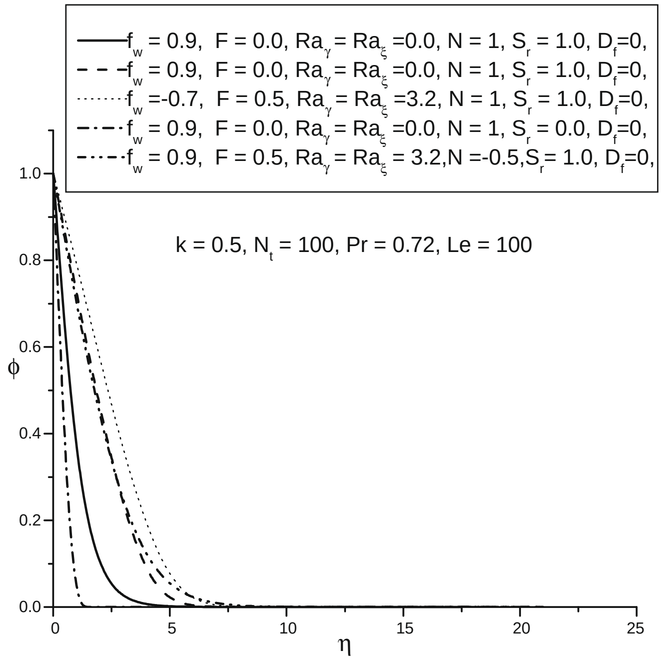

Fig. 6. Concentration distribution for zero and non zero values of dispersion, inertial, Soret, Dufour parameters.

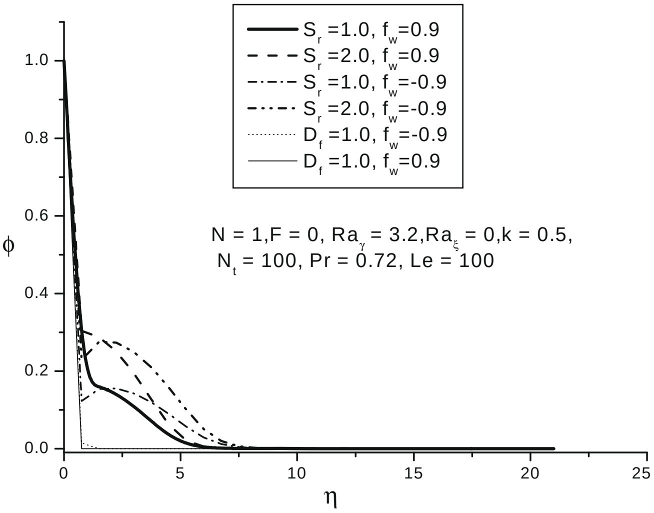

Fig. 7. Concentration distribution for various values of D_f and S_r.

Fig. 5 draws out the effect of the parameters D_f and S_r on thermal boundary layer thickness when they are zero and when they are not zero. In the aiding buoyancy as well as opposing buoyancy, when S_r is not equal to zero, the thermal boundary layer thickness is more when $f_w = -0.9$ (Fig. 5) than when $f_w = 0.9$. Further, it is evident that the effect of opposing buoyancy particularly when $f_w = -0.9$ seems to be more pronouncing compared to $f_w = 0.9$ in increasing the boundary layer thickness. In aiding and opposing buoyancy, the figure also describes that the temperature field is less effected when $S_r = 0$ and it is less dependent on the parameter D_f.

The effect of the parameters controlling inertial effect, dispersion effects, Soret and Dufour effects on concentration distribution are well presented in Figs. 6–9. From Fig. 6 we observe that the concentration boundary layer thickness widens when all the parameters are not zero. This is true in both suction as well as injection. When S_r assumes high values (Fig. 7), the concentration boundary layer thickness increases in the case of suction and injection. But the distribution of concentration is more effective in the case of injection than in the case of suction. Also we observe that the concentration distribution is weakly dependent on the parameter D_f. Fig. 8 exhibits that when Le is less, the concentration boundary layer thickness is more in suction and injection. Also, the same result that the concentration boundary layer thickness is more in the case of injection than in the case of suction is viewed.

When f_w is positive and $Le = 0.3$, the effect of opposing buoyancy is more compared to the effect of aiding buoyancy on the concentration distribution. The same is observed when f_w is negative. But the combined effect of opposing buoyancy and injection on concentration distribution is prominent from Fig. 9. The effect of Le is also seen very legibly.

The effect of suction and injection on heat and mass transfer, wall thermophoretic deposition V_{tw} under different situations are

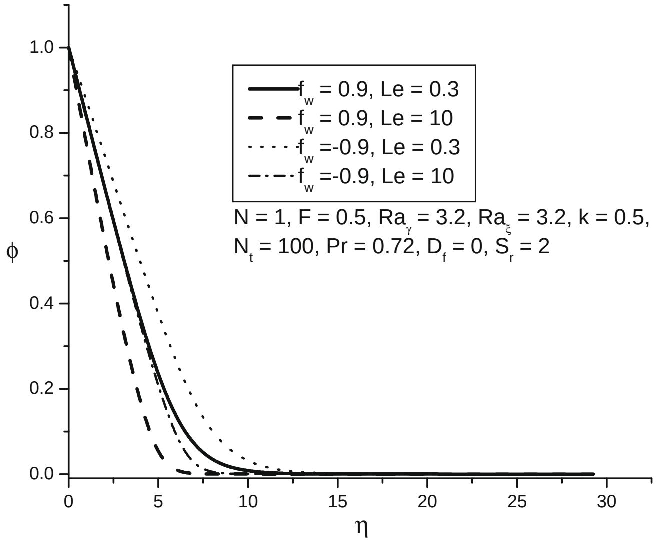

Fig. 8. Effect of *Le* on concentration distribution in the aiding buoyancy.

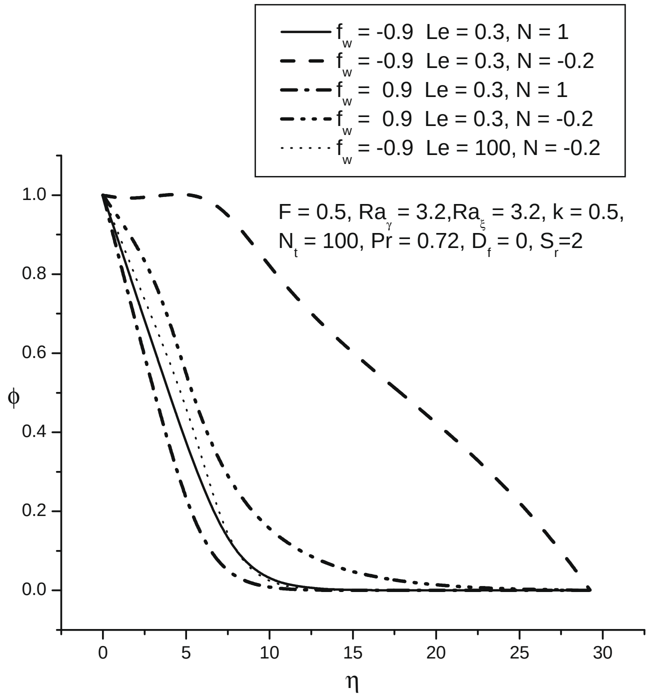

Fig. 9. Effect of *Le* on concentration distribution in the aiding and opposing buoyancy.

crystal clear from the tables. In addition to this, the effect of suction, injection on heat and mass transfer when dispersion, inertial, Soret and Dufour parameters are considered and not considered, are discussed in detail. Hence this will reflect the specialties of the second-order effects thereby establishing their own significance for determining accurately the heat transfer and mass transfer rates.

The common feature we monitor from Tables 1–3 is that heat transfer, mass transfer, and wall thermophoretic deposition velocity V_{tw} increase as we proceed from injection zone to suction zone. In both cases, suction as well as injection, magnitude of heat transfer is observed to be more when the second-order effects are considered than when they are not. But the situation is different for mass transfer and wall thermophoretic deposition velocity V_{tw}. Mass transfer from the surface to the medium and wall thermophoretic deposition velocity V_{tw} becomes less when all effects are considered than when they are not. Further, we notice that the level at which the heat transfer, V_{tw} and mass transfer varies when the second-order effects are considered will be different from that of, when they are not considered. It is clear from the tables that the Nusselt number and V_{tw} vary with marginal increase irrespective of the second-order effects being considered. But, steep increase in the case of mass transfer in the suction zone when second-order effects are not considered clearly depicts that mass transfer is more influenced due to suction only when second-order effects are not considered. Hence this clearly indicates that by neglecting the second-order effects we can not predict the heat transfer and mass transfer rates accurately.

In some situations, not only the heat and mass transfer changes considerably according to the changes in the parameter values, but also, some important facts are reflected. For instance, in Table 5 where $D_f = 1$ and $S_r = 0$, the common feature that thermophoretic velocity, heat transfer and mass transfer increases as we proceed from injection zone to suction zone is seen. But the circumstances happens to be different in Table 4 when $D_f = 0$ and $S_r = 1$. Though heat transfer and wall thermophoretic deposition velocity increases, mass transfer decreases. This establishes the importance

Table 1
Heat transfer, mass transfer and thermophoretic velocity when Dispersion, inertial effect, Soret, Dufour parameters are non-zero.

f_w	V_{tw}	$Nu_x/Ra_x^{1/2}$	$Sh_x/Ra_x^{1/2}$
0.9	0.00092379	1.2843	1.5059
0.3	0.00078812	1.0957	1.26439
−0.9	0.0005543	0.77206	0.8498

$N = 1$, $\mathrm{Pr} = 0.72$, $Le = 100$, $F = 0$, $Ra_\gamma = 3.2$, $Ra_\xi = 3.2$, $D_f = 0$, $S_r = 0$, $K = 0.5$, $N_t = 0$.

Table 2
Heat transfer, mass transfer and thermophoretic velocity when dispersion, inertial effect, Soret, Dufour parameters are zero.

f_w	V_{tw}	$Nu_x/Ra_x^{1/2}$	$Sh_x/Ra_x^{1/2}$
0.9	0.002695	0.756166	46.298281
0.3	0.001968	0.552146	18.40269
−0.9	0.001232	0.409240	1.0565

$N = 1$, $\mathrm{Pr} = 0.72$, $Le = 100$, $\mathrm{F} = 0$, $Ra_\gamma = 0$, $Ra_\xi = 0$, $D_f = 0$, $S_r = 0$, $K = 0.5$, $N_t = 100$.

Table 3
Heat transfer, mass transfer when Dispersion, inertial, Soret, Dufour, thermophoresis parameters are zero.

f_w	$Nu_x/Ra_x^{1/2}$	$Sh_x/Ra_x^{1/2}$
0.9	0.755125	46.559230
0.3	0.552034	18.5724
−0.9	0.408919	1.08312

$N = 1$, $\mathrm{Pr} = 0.72$, $Le = 100$, $F = 0$, $Ra_\gamma = 0$, $Ra_\xi = 3.2$, $D_f = 0$, $S_r = 0$, $K = 0.5$, $N_t = 0$.

Table 4
Combined effect of Ra_ξ and S_r on heat transfer, mass transfer and thermophoretic velocity.

f_w	V_{tw}	$Nu_x/Ra_x^{1/2}$	$Sh_x/Ra_x^{1/2}$
0.9	0.00356	1.000586	0.5817954
0.3	0.00283	0.794004	0.629172
0	0.00254	0.698493	0.6512
−0.3	0.00217	0.608761	0.671462
−0.9	0.00159	0.448212	0.7038251

$N = 1$, $\mathrm{Pr} = 0.72$, $Le = 100$, $F = 0$, $Ra_\gamma = 0$, $Ra_\xi = 3.2$, $S_r = 0$, $D_f = 0$, $K = 0.5$, $N_t = 100$.

Table 5
Combined effect of Ra_ξ and D_f on heat transfer, mass transfer and thermophoretic velocity.

f_w	V_{tw}	$Nu_x/Ra_x^{1/2}$	$Sh_x/Ra_x^{1/2}$
0.9	0.003515	0.986423	1.5320146
0.3	0.002681	0.78064	1.3337982
0	0.002432	0.685804	1.24321
−0.3	0.002068	0.596911	1.158433
−0.9	0.001045	0.438466	1.005718

$N = 1$, $Pr = 0.72$, $Le = 100$, $F = 0$, $Ra_\gamma = 0$, $Ra_\xi = 3.2$, $S_r = 0$, $D_f = 0$, $K = 0.5$, $N_t = 100$.

of parameter values D_f, S_r, and Ra_ξ on heat and mass transfer. Thermophoretic velocity, heat and mass transfer corresponding to $f_w = 0$ (when the wall is impermeable) is appeared to be considerably different when compared to $f_w \neq 0$.

Transport phenomenon involving the motion of small particles suspended in gaseous media and their deposition on immersed or containment solid surfaces occur often in industry and in nature. The value of the thermophoretic coefficient k depends upon the regime of the flow which results in a net force on the particle. A measure of this regime is the radius of the particle R_p, compared to the mean free path length λ. Expressions for calculating k for the flow regime $\lambda \approx R_p$ proposed by Derjaguin et al. [30] is $k = k_T \frac{(1+c_1\cdot\frac{\lambda}{R_p}\frac{k_p}{k_g})}{1+\frac{k_p}{2k_g}+c_1\cdot\frac{\lambda}{R_p}\cdot\frac{k_p}{k_g}}$ where $c_1 = 2.17$, the thermal slip coefficient, $k_T = 1.1$, and k_p, k_g are thermal conductivities of the particle and gas, respectively. Since the thermal conductivity is different for different fluids, it is obvious that the thermophoretic coefficient will be different for different fluids. Also, from the above expression it is clear that thermophoretic coefficient depends also on the radius of the particle. For SiO_2, particles of 0.1 μm radius in an O_2 background at 1300 K, we find that $k = 0.43$. Several expressions for calculating the thermophoretic coefficient k are available in the literature. In all the expressions, it is observed that k depends on the thermal conductivity of the particle k_p, thermal conductivity of the gas k_g and radius of the particle R_p. In most of the cases the value of this constant k lies between 0.25 and 1.25.

As thermophoretic constant k increases (Tables 6 and 7) wall thermophoretic deposition velocity V_{tw} increases, heat transfer increases and mass transfer decreases. These changes in the magnitude of V_{tw}, heat transfer and mass transfer will be significant in several applications.

It is evident from Tables 8 and 9 that with the increase of N_t, thermophoretic velocity V_{tw}, heat transfer decreases and mass transfer increases in both the cases $f_w = 0.9$ and -0.9. Also, for any value of N_t, heat transfer and mass transfer in suction is more than in injection. From Tables 10–12, we perceive that, increase in S_r and D_f increases the heat transfer, and thermophoretic wall deposition velocity V_{tw}. Nevertheless decreases the mass transfer.

4. Opposing buoyancy

Tables 13–15 show that heat transfer, mass transfer and wall thermophoretic deposition velocity is more in the case of suction than in the case of injection. But the scenario changes when solutal dispersion and parameter of the Soret effect is not zero. In this particular case, though heat transfer, wall thermophoretic deposition velocity increases as we proceed from injection zone to suction zone, mass transfer decreases.

Table 6
Effect of K on wall thermophoretic deposition velocity, heat transfer and mass transfer.

k	V_{tw}	$Nu_x/Ra_x^{1/2}$	$Sh_x/Ra_x^{1/2}$
0.1	0.0001846	1.283433	1.506734
0.5	0.0009237	1.28444	1.5059
1.0	0.00108	1.285455	1.5049

$N = 1$, $Pr = 0.72$, $Le = 100$, $F = 0.5$, $Ra_\gamma = 3.2$, $Ra_\xi = 3.2$, $D_f = 1$, $S_r = 0$, $N_t = 100$, $f_w = 0.9$.

Table 7
Effect of K on wall thermophoretic deposition velocity, heat transfer and mass transfer.

k	V_{tw}	$Nu_x/Ra_x^{1/2}$	$Sh_x/Ra_x^{1/2}$
0.1	0.0001107	0.7698	0.85025
0.5	0.0005539	0.77079	0.84987
1.0	0.0010069	0.7706466	0.8489

$N = 1$, $Pr = 0.72$, $Le = 100$, $F = 0.5$, $Ra_\gamma = 3.2$, $Ra_\xi = 3.2$, $D_f = 0$, $S_r = 0$, $N_t = 100$, $f_w = -0.9$.

Table 8
Effect of N_t on wall thermophoretic deposition velocity heat transfer and mass transfer.

N_t	V_{tw}	$Nu_x/Ra_x^{1/2}$	$Sh_x/Ra_x^{1/2}$
1	0.02895	0.7969203	0.7776041
10	0.005244	0.7941354	0.7896358
100	0.0005708	0.7936448	0.7924356

$N = 1$, $Pr = 0.72$, $Le = 100$, $F = 0.5$, $Ra_\gamma = 3.2$, $Ra_\xi = 3.2$, $S_r = 1.0$, $K = 0.5$, $f_w = -0.9$.

Table 9
Effect of N_t on wall thermophoretic deposition velocity heat transfer and mass transfer.

N_t	V_{tw}	$Nu_x/Ra_x^{1/2}$	$Sh_x/Ra_x^{1/2}$
1	0.04795	1.32024	1.273344
10	0.008695	1.316565	1.30587
100	0.0009465 2	1.315931	1.312759

$N = 1$, $Pr = 0.72$, $Le = 0.3$, $F = 0.5$, $Ra_\gamma = 3.2$, $Ra_\xi = 3.2$, $D_f = 0$, $K = 0.5$, $f_w = -0.9$, $N_t = 100$.

Table 10
Effect of $S_r$on wall thermophoretic deposition velocity, heat transfer and mass transfer.

S_r	V_{tw}	$Nu_x/Ra_x^{1/2}$	$Sh_x/Ra_x^{1/2}$
0.1	0.0089995	1.3621	1.02023
0.9	0.00902	1.3758	0.899
3.0	0.009310	1.40968	0.66544

$N = 1$, $Pr = 0.72$, $Le = 0.3$, $F = 0.5$, $Ra_\gamma = 3.2$, $Ra_\xi = 3.2$, $D_f = 0$, $K = 0.5$, $f_w = -0.9$, $N_t = 100$.

Table 11
Effect of S_r on wall thermophoretic deposition velocity heat transfer and mass transfer.

S_r	V_{tw}	$Nu_x/Ra_x^{1/2}$	$Sh_x/Ra_x^{1/2}$
0.1	0.00546	0.8279164	0.68022
0.9	0.005534	0.8381097	0.64347
3.0	0.00591	0.8633	0.544121

$N = 1$, $Pr = 0.72$, $Le = 0.3$, $F = 0.5$, $Ra_\gamma = 3.2$, $Ra_\xi = 3.2$, $S_r = 0$, $K = 0.5$, $f_w = 0.9$, $N_t = 100$.

Table 12
Effect of D_f on wall thermophoretic deposition velocity heat transfer and mass transfer.

D_f	V_{tw}	$Nu_x/Ra_x^{1/2}$	$Sh_x/Ra_x^{1/2}$
0.1	0.0089877	1.36088	1.03217
0.9	0.009136	1.3652225	1.030549
3.0	0.009145	1.3766547	1.0298993

Opposing buoyancy: $N = -0.5$, $Pr = 0.72$, $Le = 100$, $F = 0$, $Ra_\gamma = 3.2$, $Ra_\xi = 0$, $S_r = 1$, $D_f = 0$, $K = 0.5$, $N_t = 100$.

Table 13
Effect of thermal dispersion in the opposing buoyancy on wall thermophoretic deposition velocity, heat transfer and mass transfer when $S_r = 1$.

f_w	V_{tw}	$Nu_x/Ra_x^{1/2}$	$Sh_x/Ra_x^{1/2}$
0.9	0.001343	0.9804002	24.476094
0.3	0.001069	0.7797816	5.569588
0	0.0008939	0.6519422	0.621526
−0.2	0.0008239	0.600089	0.391843

$N = -0.5$, $\Pr = 0.72$, $Le = 100$, $F = 0$, $Ra_\gamma = 3.2$, $Ra_\xi = 0$, $S_r = 1$, $D_f = 0$, $K = 0.5$, $N_t = 100$.

Table 14
Effect of thermal dispersion in the opposing buoyancy on wall thermophoretic deposition velocity, heat transfer and mass transfer when $D_f = 1$.

f_w	V_{tw}	$Nu_x/Ra_x^{1/2}$	$Sh_x/Ra_x^{1/2}$
0.9	0.001975	1.4405508	45.596882
0.3	0.001488	1.08595	16.785126
−0.3	0.000889	0.6486038	0.019996

$N = -0.5$, $\Pr = 0.72$, $Le = 100$, $F = 0$, $Ra_\gamma = 0$, $Ra_\xi = 0$, $S_r = 0$, $D_f = 1$, $K = 0.5$, $N_t = 100$.

Table 15
Effect of solutal dispersion in the opposing buoyancy on wall thermophoretic deposition velocity, heat transfer and mass transfer when $S_r = 1$.

f_w	V_{tw}	$Nu_x/Ra_x^{1/2}$	$Sh_x/Ra_x^{1/2}$
0.9	0.0021038	0.590259	0.053435
0.3	0.001298	0.364272	0.08066
−0.3	0.000696	0.195444	0.116129

$N = -0.5$, $\Pr = 0.72$, $Le = 100$, $F = 0$, $Ra_\gamma = 0$, $Ra_\xi = 0$, $S_r = 1.0$, $D_f = 0$, $K = 0.5$, $N_t = 100$.

Thus the analysis reveals that the rate of heat transfer and mass transfer varies due to the complex interaction among several parameters. Hence the rate of change of heat transfer, mass transfer and wall thermophoretic deposition velocity varies depending on Soret, Dufour, dispersion parameter values and thermophoretic coefficient k. Thus the combined effect of all parameters is more important in determining more accurately the heat transfer and mass transfer rates.

5. Conclusions

Similarity solution technique is used to analyze the thermophoresis effect on a vertical plate embedded in a non-Darcy porous medium under the influence of Dufour and Soret effects in suction as well as injection. The effect of the parameters D_f and S_r on flow field, temperature distribution, concentration distribution, thermophoretic wall deposition velocity, heat transfer and mass transfer is analyzed for different values of dispersion parameters and Lewis number. It is worth mentioning that the parameter S_r is more influential than the parameter D_f on the convective transport. As the value of S_r increases, the flow field boundary layer thickness, thermal boundary layer thickness and concentration boundary layer thickness increases. This rate of increase is more in the case of injection (in the aiding buoyancy as well as opposing buoyancy) in contrast with suction. But this is highlighted to be more effective under the combined influence of opposing buoyancy and injection.

In both the cases (suction and injection) magnitude of heat transfer is observed to be more when the second-order effects are considered than when they are not. But the situation is different for mass transfer and wall thermophoretic deposition velocity V_{tw}. In the case of suction and injection, Mass transfer and wall thermophoretic deposition velocity V_{tw} becomes less when all effects are considered. Usually, heat transfer, mass transfer, wall thermophoretic deposition velocity increases as we proceed from injection zone to suction zone. But in some situations (depending on the values of the parameters) it is seen that though heat transfer and wall thermophoretic deposition velocity increases as we proceed from injection zone to suction zone, mass transfer decreases. This echoes the importance of parameter values of D_f and S_r, and dispersion parameters on heat and mass transfer. The thermophoretic coefficient k is different for different fluids. It also varies as the properties of the fluids change. Hence the concentration distribution, heat and mass transfer rate changes. As thermophoretic constant k increases, wall thermophoretic deposition velocity V_{tw} and heat transfer increases whereas mass transfer decreases.

Thus the rate of change of heat transfer, mass transfer and wall thermophoretic deposition velocity varies depending on Soret, Dufour, dispersion parameters and thermophoretic coefficient k. Thus the combined effect of all parameters is important in determining more accurately the heat transfer and mass transfer rates.

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